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Fractal strain distribution and its implications for cross-section balancing: Reply

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INTRODUCTION

The major questions raised by Dunne & Ferrill (1995) are: (a) does strain have a fractal distribution, and (b) how does shortening in the blind duplex balance with shortening in the roof sequence at the Cacapon Mountain anticlinorium.

FRACTAL STRAIN DISTRIBUTION

Dunne and Ferrill agree that strain does appear at all scales and that it is important to consider all scales when balancing a cross-section. That strain appears at all scales is not a new idea. Geologists used synclinorium and anticlinorium to describe the scale-independent feature of folds long before Mandelbrot (1967) invented the word 'fractal'. I think the fractal method can provide a quantitative tool for describing the distribution of strain at different scales. In my original paper, comparing $D = 1.001$ in the Cambrian–Ordovician (CO) carbonates to $D = 1.043$ in the Devonian–Silurian (DS) rocks, I made the valid statement that not much deformation has occurred in the CO carbonates at smaller scales as compared to that in the DS roof sequence. This does not eliminate the possibility that grain-scale shortening could have magnitudes of 10–15% farther to the east (Cloos 1971, Dean 1972, Kulander 1972, Evans & Dunne 1991), and the impact of this strain in the CO carbonates is discussed in more detail in the original paper (Wu 1993, pp. 1505–1506). If the same 10–15% strain occurred in the CO carbonates at the Cacapon Mountain duplex, it would further enhance my conclusion. The 10–15% additional strain has to be balanced in the roof sequence at smaller scales; otherwise, this amount of shortening will be transferred to the foreland, which is unlikely.

Dunne and Ferrill are correct in pointing out that geological structures are three-dimensional. The fractal dimension of a geological surface should be between two and three. As geoscience and computer technology advance, geologists are starting to work on three-dimensional structures using three-dimensional technology. In the meantime, two-dimensional cross-section analysis will be be needed and cross-section validation techniques will remain relevant. As long as the assumptions for drawing cross-sections are viable (such as plane strain, section line oriented along the displacement direction, etc.), fractal analysis can be applied under the same assumptions. In the Cacapon Mountain anticlinorium, it seems reasonable to predict that a fractal surface will reveal a fractal dimension of greater than but close to two for the duplex of CO carbonates and larger fractal dimensions ($2 < D < 3$) for the upper Ordovician, Devonian and Silurian units in the roof sequence.

Dunne and Ferrill are concerned that the base of the Silurian Bloomsburg Formation (DS) is not representative of the majority of the overlying Devonian sequence. The answer to this question really depends on what is the primary problem being addressed. The paper is trying to explain the shortening discrepancy between the roof sequence and the duplex of CO carbonates. In terms of strain distribution, DS has a larger fractal dimension, which indicates that more deformation occurred at smaller scales. If the primary concern is the behaviour of individual units in the roof sequence, then indeed these units are different. In the original study fractal dimensions using the compass method were calculated for most units in the roof sequence and the differences between them are in the order of 0.01. The numbers indicated that deformation is distributed differently for individual units in the roof sequence. As noticed by Dunne & Ferrill (1995), these differences are even apparent on the cross-section; some units have more folds and some have more faults at different scales. However, the general results of the analysis will not change if any other units in the roof sequence are used.

I agree with Dunne & Ferrill (1995) that more tests are needed to prove or disprove that a fractal strain distribution is the right explanation for the shortening discrepancy between rock units of different physical properties.

CROSS-SECTION BALANCING

A flexural slip restoration in fig. 1. of Dunne and Ferrill shows that the length at the base of the unit

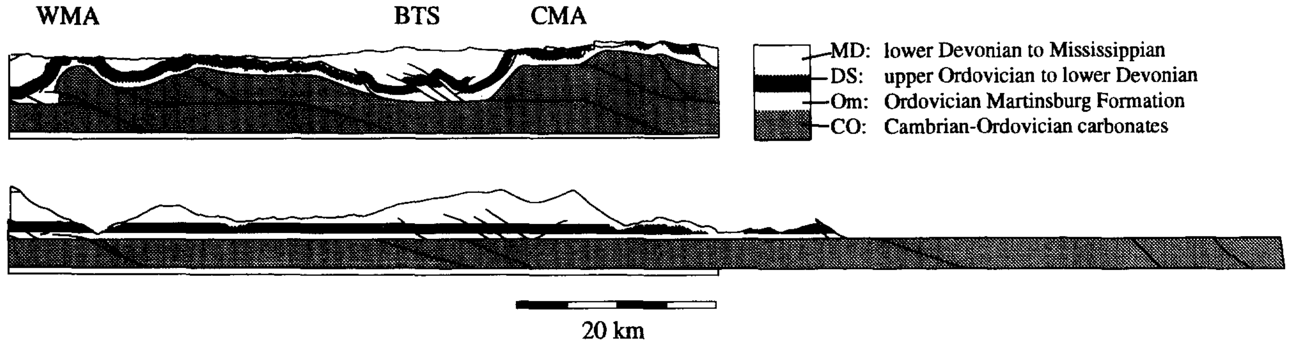

Fig. 1. The cross-section presented by Dunne & Ferrill (1995) across the Valley and Ridge province in central Appalachians and its restoration by flexural-slip algorithm (using GeoSec^{TM}) with both bed-length and section area conserved. The erosional portion of profile at the base of the unit DS has been extrapolated. The length of the deformed state section is 70.80 km. The length of base DS in the restored state is 82.28 km. The length of the blind thrust sheets of CO carbonates in the restored state is 126.52 km. There are 44.24 km more shortening in the CO than in the DS. 20 km of the 44.24 km could have been transferred into the Appalachian Plateau, and 10–14 km could have been accommodated by penetrative layer-parallel shortening strain. Greater than 10 km shortening remain unexplained. This calculation does not consider the displacement transferred from North Mountain thrust from the east.

labeled DS in the restored state is 82.28 km and the length of the CO carbonates in the restored state is 126.52 km (Fig. 1). The length of the deformed state section is 70.80 km.

Dunne and Ferrill suggest that Evans (1989) has shown that the North Mountain thrust did not transfer +60 km of displacement into the roof sequence above the Martinsburg detachment in the Valley and Ridge province. They believe that this displacement was transferred along with the North Mountain thrust sheet which used to cover most of the Valley and Ridge province. However, based on abundant field evidence and the work of others in the area, Dean *et al.* (1990, p. 1443) argued that "significant westward translation of the cover above the Martinsburg detachment must have occurred to accommodate the magnitude of the emplacement of the North Mountain sheet". Geometrically, if no displacement was transferred from the North Mountain thrust fault into the roof sequence, there would be no Martinsburg detachment in front of the North mountain fault, and there would be no shortening in the roof sequence, in the east part of the Cacapon Mountain anticlinorium. As indicated by fig. 1 in Dunne & Ferrill (1995), the Martinsburg detachment is there and the roof sequence at this part of the structure, is folded. In addition, the northern tip point of the North Mountain thrust fault in northern Washington County, Maryland is less than 50 km from the cross-section location (fig. 1 in Dunne & Ferrill 1995). For a +60 km displacement to diminish in less than 50 km along a strike requires an abnormal fault displacement gradient.

Although I think that to assume that no displacement was transferred from the North Mountain thrust into the roof sequence in the Valley and Ridge province is kinematically inadmissible and geologically incorrect, to make the problem simple, I will not consider any displacement transferred from the North Mountain thrust in the following calculation. The restoration (Fig. 1) indicates that there is 44.24 km more net shortening in the CO carbonates than in the roof sequence above the Martinsburg detachment. As suggested by Dunne & Ferrill (1995), the Plateau rocks in front of the Willis Mountain anticline could have accommodated 20 km of displacement transferred from the Valley and Ridge province; a 15–20% layer-parallel shortening strain (Ferrill & Dunne 1989) gives 10–14 km net shortening. There is still a +10 km displacement discrepancy remaining. There are several possible explanations for the discrepancy, such as a large quantity of material dissolution/transport, part of the duplex developed in the Taconic orogeny, etc. Again, I propose a fractal strain distribution for a possible explanation.

It is clear from the cross-section presented in Fig. 1 in Dunne & Ferrill (1995) that the profiles of the CO duplexes are simple: there are fewer faults and no folds at smaller scales. Furthermore, the profiles of the upper Ordovician, Silurian, and Devonian units above the Martinsburg detachment are more complicated and have more small-scale folds and faults. Fractal dimensions provide a quantitative measurement of this complexity (Fig. 2). Using the compass method, fractal dimensions of 1.01 were determined for CO carbonates and 1.04 for the profile at the base of DS (Figs. 1 and 2). The smallest fold wavelength on a cross-section of 70 km, which can be displayed effectively on a page (fig. 1 in Dunne & Ferrill 1995), is about 1 km (~2 mm on the page). The smallest ruler (r) used for measurement is 600 m (~1 mm on the page, Fig. 2). Any structures less than 1 km in scale cannot be measured effectively. Comparing these measurements to the previous ones (Wu 1993), the fractal dimensions for DS in the roof sequence are virtually the same (~1.04), but the $D = 1.01$ for the CO carbonates in the section of Dunne and Ferrill is much larger than $D = 1.001$ for the same units in the section of Dean *et al.* (1985) (Wu 1993). The reason for this difference is that on the regional section across the entire Valley and Ridge province in Dunne and Ferrill, the structures of the CO carbonates are more complicated (more folds and faults) than that in the smaller section across the Capacon Mountain anticlinorium in Dean *et al.* (1985), fig. 4 in Wu (1993). Neverthless, $D = 1.04$ for the DS indicates that more

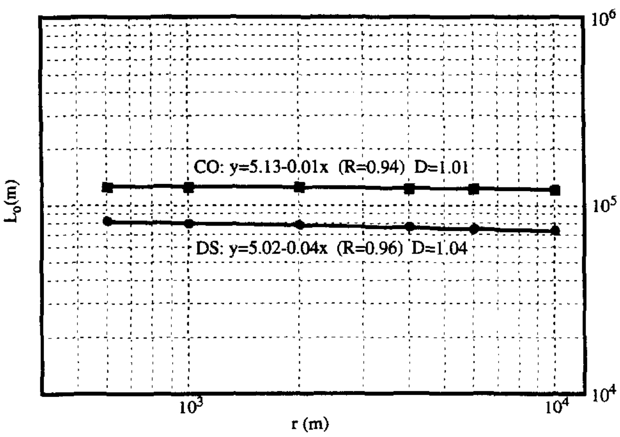

Fig. 2. By using a compass method, the fractal dimension was estimated at $D = 1.01$ for the thrust sheets in the CO carbonates, and $D = 1.04$ for the roof sequence of the upper Ordovician, Silurian and Devonian rocks. More strain distributed at smaller scales in DS than in CO. The smallest folds in fig. 1 of Dunne & Ferrill (1995) displayed with confidence is around 1000 m (~2 mm in fig. 1). The smallest ruler used is 600 m (~1 mm in fig. 1 of Dunne & Ferrill 1995).

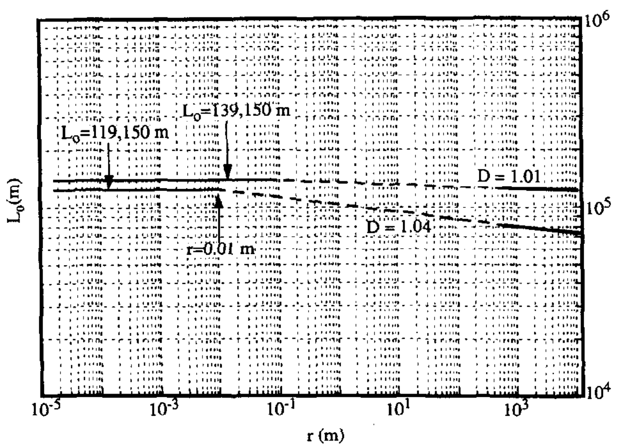

Fig. 3. If the fractal dimensions are linearly extrapolated toward the small scale. To estimate the bed length of CO carbonates in their undeformed state, a 5% layer-parallel shortening strain gives $L_0 = 139.15$ km (Fig. 3). Assuming 20 km of shortening was transferred in the Appalachian Plateau, the bed length in the roof sequence should be 119.15 km. The horizontal line $L_0 = 119.15$ km intersects the line of $D = 1.04$ at about $r = 1$ cm. If a ruler of centimeter scale is used to measure the total length, the roof sequence should yield an amount of shortening which can balance the total amount of shortening in the blind thrusts of the CO carbonates.

strain is distributed at smaller scales in the roof sequence, and less strain is distributed at smaller scales in the CO carbonates.

If we can discount other possibilities, such as +10 km of additional fault displacement transferred into the Plateau, +10 km of displacement transferred along some unknown back thrust, or a large body of materials with +10 km in section length dissolved and transported to some unknown place, etc., then the +10 km (+14%) shortening must have been accommodated by deformation occurring at smaller scales (between ~mm and <km, Fig. 3). In Fig. 3, the measured fractal dimensions from the cross-section (thick lines) were linearly extrapolated to a fine scale (dashed lines). To estimate the bed length of CO carbonates in their undeformed state, a 5% layer-parallel shortening strain gives $L_0 = 139.15$ km (Fig. 3). Assuming 20 km of shortening was transferred in the Appalachian Plateau, the bed length in the roof sequence should be 119.15 km. The horizontal line $L_0 = 119.15$ km intersects the line of $D = 1.04$ at about $r = 1$ cm (Fig. 3). This intersection predicts that if a ruler of centimeter scale is used to measure the total length, units in the roof sequence should yield an amount of shortening which can balance the total amount of shortening in the blind thrusts in the CO carbonates.

This calculation does not include any displacement transferred from the North Mountain thrust fault. Based on surface and subsurface data, Dean *et al.* (1990, pp. 1442–1443) pointed out that the structural complexity of the roof sequence in the Valley and Ridge province is much greater than is generally acknowledged. Dunne and Ferrill argue that the fifth-order fold (fig. 10 in Wu 1993) developed only in uncleaved units and consequently the shortening is already counted in the penetrative layer-parallel strain. If this is true, small folds and faults of other orders, either missing or under represented by the regional cross-section (Fig. 1), must have accommodated the +10 km shortening at the cross-section scale plus the amount of layer-parallel shortening in the CO carbonates.

The outcrop location in fig. 10 in Wu (1993) is 35 m from the east end of the Roundtop cut on the abandoned Western Maryland railroad, 5 km west of Hancock, Maryland [station IIb in Geiser (1970) and near Sample Site D in Geiser (1974)]. As indicated in the figure, small folds and spaced cleavage coexist (seven fold hinges were measured, cleavage spacing is around 10 cm and average orientation is 025°/49°E). In general, units in the roof sequence are less competent than the massive CO carbonates, but individual units have different physical properties, such as layer thickness, grain size, lithology, etc. I do not expect that the folds and faults were distributed evenly among the different units.

CONCLUSIONS

In summary, both cross-sections presented by Dean *et al.* (1985) (Wu 1993) and Dunne and Ferrill in the discussion have strain distributed at all scales. If the ruler used to measure the total shortening is small enough, the shortening in the roof sequence can be balanced with that in the Cambrian–Ordovician carbonates. Displacement balance between the roof sequence and the blind thrusts is misinterpreted as 'local compensation' by Dunne and Ferrill. Fractal analysis is a useful method to quantify how much strain has occurred at smaller scales.

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